

# An Optimal Adaptive Power System Stabilizer

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**Abstract** - An optimal control algorithm with adaptive system parameters and state variables estimation is presented in this paper. The optimal control algorithm is calculated by solving the algebraic Riccati equation of the linearized closed loop system model obtained by using an adaptive recursive least squares identification algorithm. The feedback control is achieved by recalculating the control sequence each sampling period. An application of the algorithm as a power system stabilizer is illustrated.

**Key Words** - optimal control; adaptive control; RLS algorithm; Kalman filter; LQG regulator; power system stabilizer; state-space estimation

## 1 Introduction

A power system is a sophisticated combination of multiple electrical and mechanical components. In general, these elements are highly nonlinear. Also, power system operation is characterized by a wide range of operating conditions, random load changes and various unpredictable disturbances.

The application of adaptive control theory has been studied widely in recent years [1]-[4]. The principal attraction of this approach is that the adaptive controller can track any changes in the plant parameters using identification techniques. The adaptive-control design technique based on an explicit identification of the transfer function of the plant to be controlled, using an adaptive recursive least squares (RLS) identification algorithm, is considered in this paper. The optimal control is calculated by solving the algebraic Riccati equations for the identified closed-loop system model obtained by the RLS identification algorithm. The control is calculated each sampling period.

The proposed controller has been implemented as a power system stabilizer on a digital signal processor (DSP) and tested on a physical model of a single machine power system. Real-time test results given in the paper illustrate the effectiveness of the proposed controller.

## 2 Controller Structure

The adaptive-control design considered in this paper is based on an explicit identification of the transfer function between the system input  $u(nT)$  and output  $y(nT)$  as shown in Fig. 1. The system output is sampled at discrete intervals of time  $T$  and modeled with an ARMA difference equation of the form

$$\hat{y}(nT) = \sum_{i=1}^m b_i u((n-i)T) - \sum_{i=1}^m a_i y((n-i)T) \quad (1)$$

where  $\hat{y}(nT)$  is the one step ahead prediction of the system output,  $m$  is the model order, and the coefficients  $a_i$   $b_i$  are the model parameters. The Recursive-Least-Squares (RLS) adaptive technique [5] is used for the transfer-function estimation. The parameters should be selected in such a way that the cost function for the RLS algorithm

$$J_{RLS}(n) = \sum_{i=1}^n \lambda^{n-i} |e_{RLS}(i)|^2 \quad \lambda = e^{-1/l} \quad (2)$$

is minimum, where  $\lambda$  the forgetting factor, is close to, but less than, 1.0 [6],  $l$  is the memory length measured in samples and  $e$  is the natural logarithm base. The use of a forgetting factor is intended to ensure that data points in the distant past are "forgotten" in order to afford the possibility of following the statistical variations of the observable data when the adaptation operates in a non-stationary environment [6].

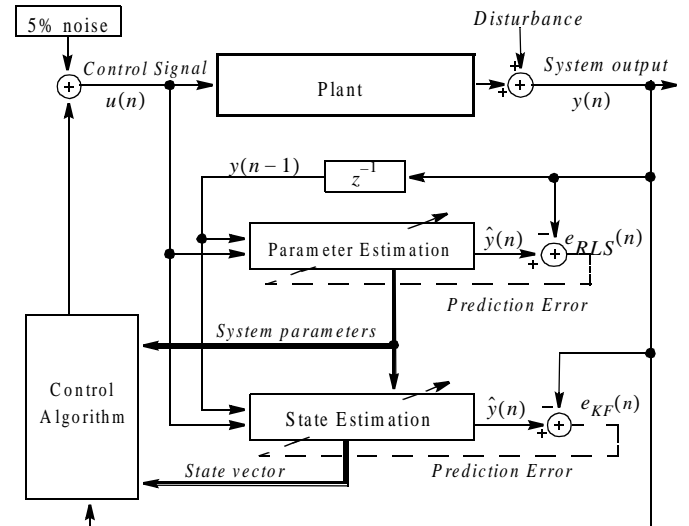


Fig. 1 Adaptive Optimal Controller Structure

The dynamic system in state-space representation can be described, based on the estimated system parameters  $a_i$  and  $b_i$ , by the process equation (3), and by the measurement equation (4) below:

$$x((n+1)T) = A(nT)x(nT) + B(nT)u(nT) + G(nT)w(nT) \quad (3)$$

$$y(nT) = C(nT)x(nT) + v(nT) \quad (4)$$

where

$$A(nT) = \begin{bmatrix} -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & \dots & 0 \\ -a_3 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ -a_{m-1} & 0 & 0 & \dots & 1 \\ -a_m & 0 & 0 & \dots & 0 \end{bmatrix}, B(nT) = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ \dots \\ b_m \end{bmatrix} \quad (5)$$

$$G(nT) = [g_1, g_2, \dots, g_m]^T \quad (6)$$

$$C(nT) = [1, 0, 0, \dots, 0].$$

The state-space vector,  $x(nT)$ , is estimated with Kalman Filter, and  $g_i = 1$ . It is necessary to define strict requirements for the noise quantities  $w(nT)$  and  $v(nT)$  in equations (3) and (4) respectively. The process noise  $w(nT)$  represents disturbances or modeling inaccuracies. It is assumed that it is a white noise process with zero mean, and its covariance is defined by

$$E\{w(kT)w(iT)\} = \begin{cases} Q_K(kT) & i = k \\ 0 & i \neq k \end{cases} \quad (7)$$

where  $E\{\circ\}$  is the statistical expectation, and  $Q_K(kT)$  is the process noise covariance [7]. The measurement noise  $v(nT)$  is due to sensor inaccuracy, and is assumed to be a white noise process with zero mean. Its covariance matrix is defined by

$$E\{v(kT)v(iT)\} = \begin{cases} R_K(kT) & i = k \\ 0 & i \neq k \end{cases} \quad (8)$$

where  $R_K(kT)$  is the measurement noise covariance [7].

The Kalman filter is based on a cost function minimization process [8]

$$J_K(n) = E[e_{KF}^2(nT)]. \quad (9)$$

In the method used here, it is proposed to adapt the Kalman filter to the changes in the process noise  $w(nT)$  and the measurement noise  $v(nT)$  statistics.

The optimal control is computed by using the identified model parameters and estimated states of the system. Based on linear control theory [9], the quadratic performance index used is

$$J_C = \sum_{k=0}^{N-1} x^T((n-k)T)Q_C(kT)x((n-k)T) + u^T((n-k)T)R_C(kT)u((n-k)T) \quad (10)$$

where  $N$  is finite,  $Q_C(k)$  is symmetric positive semi-definite and  $R_C(k)$  is positive definite. With this algorithm both the plant and the cost-function matrices are allowed to be time varying. The design results in a control law of the form

$$u((n+1)T) = -K_C(nT)x(nT) \quad (11)$$

which is linear, time-varying, full state feedback. The solution for  $K_C(nT)$  can be determined with the Algebraic Riccati Equation [8].

### 3 System Parameter Estimation

The simplest way to implement and describe the RLS algorithm is to use matrix notation and calculation. The RLS algorithm estimates the parameter vector

$$\Theta(nT)^T = [b_1 \dots b_m \ a_1 \dots a_m \ c_1 \dots c_k]. \quad (12)$$

At the start of each recursion the measurement vector

$$\Psi(nT)^T = [u(nT), \dots, u((n-m+1)T), -y((n-1)T), \dots, y((n-m)T), \hat{e}((n-1)T), \dots, \hat{e}((n-k)T)] \quad (13)$$

is updated with the measurement variables  $u(nT)$ ,  $y((n-1)T)$ .

For the RLS algorithm the recursion is initialized by choosing a starting value  $P(0)$  that assures the nonsingularity of the correlation matrix  $P(0)^{-1}$ . It is recommended that it should be initialized as

$$P(0) = \delta I \quad (14)$$

where  $I$  is an  $m$ -by- $m$  identity matrix, and  $\delta$  is a large positive constant (for example  $\delta = 10^{12}$  is used here). If the selected constant  $\delta$  is relatively small, the convergence of identification at the start is much slower. The rest of the vectors should be initialized to zero or updated by the measurement variables.

Equations for the RLS algorithm are [5]:

$$e(nT) = y(nT) - \Psi(nT)^T \Theta((n-1)T) \quad (15)$$

$$K(nT) = \frac{P((n-1)T)\Psi(nT)}{\lambda + \Psi(nT)^T P((n-1)T)\Psi(nT)} \quad (16)$$

$$P(nT) = \frac{1}{\lambda} (P((n-1)T) - K(nT)\Psi(nT)^T P((n-1)T)) \quad (17)$$

$$\Theta(nT) = \Theta((n-1)T) + K(nT)e(nT)\beta(nT) \quad (18)$$

The least-squares estimate is optimal if the disturbances are Gaussian and this causes the prediction error to appear as white noise. In practice, the least-squares estimates have some

drawback because the assumptions are violated. It is a direct consequence of the least-squares formulation that a single large error will have a drastic influence on the result because the errors are squared in the criterion [6]. To correct this impact, a tracking constrained coefficient,  $\beta(nT)$  [4] is introduced in the algorithm:

$$\beta(nT) = \begin{cases} 1 & \text{if } N_2/N_1 \leq \beta_0 \\ \beta_0/N_2 & \text{if } N_2/N_1 > \beta_0 \end{cases} \quad (19)$$

$$N_1 = \|\Theta((n-1)T)\|_1 \quad N_2 = \|K(nT)e(nT)\|_1 \quad (20)$$

and the norm is defined as

$$\|X\|_1 = \sum_{i=1}^{\dots} |x_i|. \quad (21)$$

The correction term in equation (18) is proportional to the prediction error,  $e(nT)$ , modified by the **tracking constrained coefficient**  $\beta(nT)$ . The net effect of this weighting is to decrease the impact of large errors, by which the RLS algorithm becomes robust. The  $\beta_0$  variable is selected empirically as an appropriate threshold to eliminate the influence of large disturbances. However, it is very important that in the first period, when the adaptation is starting after the initialization, the tracking constrained coefficient be disabled by setting  $\beta(nT) = 1$ . Otherwise, the adaptation will be disabled by the action of this constraint.

#### 4 System State Estimation

The Kalman filter consists of two parts; the time update equations (22) and (23), which take into account the error in modeling the system dynamics, and the measurement update equations (24), (25) and (26), which take into account the effect of error in the measurement of the system output. In other words, the discrete Kalman filter has two steps at each sampling time  $nT$ ; the first is the time update or a prior update, by which the state vector,  $\hat{x}((n-1)T)$ , is updated to  $\hat{x}^-(nT)$ , and the other is the measurement update, by which the measurement  $y(nT)$  at time  $n$  is incorporated to provide the updated estimate  $\hat{x}(nT)$  [9].

Time update equations:

$$P_{\bar{K}}(nT) = A(nT)P((n-1)T)A^T(nT) + G(nT)Q_K(nT)G^T(nT) \quad (22)$$

$$\hat{x}^-(nT) = A(nT)\hat{x}((n-1)T) + B(nT)u((n-1)T) \quad (23)$$

Measurement update equations:

$$K_K(nT) = P_{\bar{K}}(nT)C^T(nT)[C(nT)P_{\bar{K}}(nT)C^T(nT) + R_K(nT)]^{-1} \quad (24)$$

$$P_K(nT) = [I - K_K(nT)C(nT)]P_{\bar{K}}(nT) \quad (25)$$

$$x(nT) = x^-(nT) + K_K(nT)y(nT) - K_K(nT)C(nT)x^-(nT) \quad (26)$$

The recursive Kalman Filter, described above, is developed with certain assumptions about the system's mathematical model. These assumptions include complete information about noise statistics  $Q_K(nT)$  and  $R_K(nT)$ . In practice, this information is usually not completely known. The role of the noise covariances,  $Q_K(nT)$  and  $R_K(nT)$ , in the Kalman Filter is to adjust the Kalman gain in such a way that it controls the filter "bandwidth" as the state and the measurement errors vary [10]. By using a curve-fitting method, it is possible to get an estimate of these covariances.

**Estimation of the Measurement Noise Covariance** - The idea of this estimation is based upon the measurement equation (4). Rearranging this equation leads to the definition of the measurement noise

$$v(nT) = y(nT) - C(nT)\hat{x}(nT). \quad (27)$$

In equation (27),  $v(nT)$  represents the residual at time  $nT$ .  $R_K(nT)$ , can be estimated using the  $N$  most recent residuals [11], as

$$\hat{R}_K(nT) = \frac{1}{N-1} \sum_{i=0}^{n-1} [v((n-i)T) - \bar{v}]^2 \quad (28)$$

$$\bar{v} = \frac{1}{N} \sum_{i=0}^{n-1} v((n-i)T). \quad (29)$$

**Estimation of the State Noise Covariance** - For the estimation of  $Q_K(nT)$ , the residual estimation error  $e_{RLS}(nT)$ , as obtained in the adaptive RLS algorithm, could be used. The state-space representation of the system model is a modified representation of that algorithm. This logically leads to the assumption that the prediction error  $e_{RLS}(nT)$  and the process noise  $w(nT)$  represent the same characteristics of the model - the error of the model. By using the prediction error  $e_{RLS}(nT)$  of the adaptive algorithm as an "estimate" of the error in the system model, the estimate of the state noise covariance can be obtained with the equation

$$\hat{Q}_K(nT) = \frac{1}{N-1} \sum_{i=0}^{n-1} [e_{RLS}((n-i)T) - \bar{e}_{RLS}]^2 \quad (30)$$

$$\bar{e}_{RLS} = \frac{1}{N} \sum_{i=0}^{n-1} e_{RLS}((n-i)T). \quad (31)$$

#### 5 Optimal Control

Solutions for  $K_C(nT)$ , in equation (10), can be expressed with the Algebraic Riccati Equation [8], and can be obtained by the following recursive equation

$$K(nT) = [B^T(nT)P((n-1)T)B(nT) + R_C(nT)]^{-1} B^T(nT) \quad (32)$$

$$P((n-1)T)A(nT)$$

where  $P_C(nT)$  is the solution of the algebraic Riccati equation

$$P_C(nT) = A^T(nT)P_C((n-1)T)[A(nT) - B(nT)K(nT)] + Q_C(nT). \quad (33)$$

The recursion is started with the initial values  $P_C(0) = Q_C(0)$  and  $K(0) = 0$ .

To say a control system is “optimal” means only that the control law minimizes a given performance index. In the case of linear quadratic Gaussian control the system performance depends on the weighting matrices  $Q_C(k)$  and  $R_C(k)$  [12]. In the design, a trade-off must be made between control activities and output performance. Trade-off studies are made by repeatedly varying the  $Q_C(k)$  and  $R_C(k)$  weight parameters in the selected performance index (10).

The initial guess of weights can be made using the following rule of thumb [13]. If the specifications are given in terms of the maximum allowed deviations in the states and the control signals for a given disturbance, the weights, can be selected as

$$Q_C = \text{diag}(q_1, q_2, \dots, q_m) \quad (34)$$

$$R_C = \text{diag}(r_1, r_2, \dots, r_p) \quad (35)$$

where  $m$  is the order of the system and  $p$  is the number of control signals. In this work  $p=1$ . The diagonal elements can be defined according to a rule of thumb [6],[12],[13] as

$$q_i = \frac{1}{[(x_i)_{\max}]^2} \quad r_i = \frac{1}{[(u_i)_{\max}]^2} \quad (36)$$

where the index  $\max$  selects the maximum allowed deviation in the state or control signal for possible disturbances.

## 6 Experimental Studies

The behavior of the proposed optimal adaptive control system, OAPSS, has been investigated on a physical model of a single machine infinite bus power system under different operating conditions and disturbances. The model is available in the Power System Research Laboratory at the University of Calgary and consists of a 3-phase 3kVA micro-synchronous generator connected to an infinite busbar through a double circuit transmission line model and the automatic voltage regulator. In the power system model a commercial automatic voltage regulator made by ABB of Switzerland is used. This is the PHSC2 Programmable Logic Controller (PLC), specially designed for power systems.

The control algorithm is implemented on a single board computer, which uses a Texas Instruments TMS320C30 digital signal processor (DSP) to provide the necessary computational power, which is a 32 bit floating-point device. The digital signal processor board is installed in a personal computer with the corresponding development software and debugging application program. The analog to digital (A/D) input channel of DSP board receives the  $P_e$  signal, sampled at a 50 ms interval, calculates the control signal  $U_{PSS}$  which is

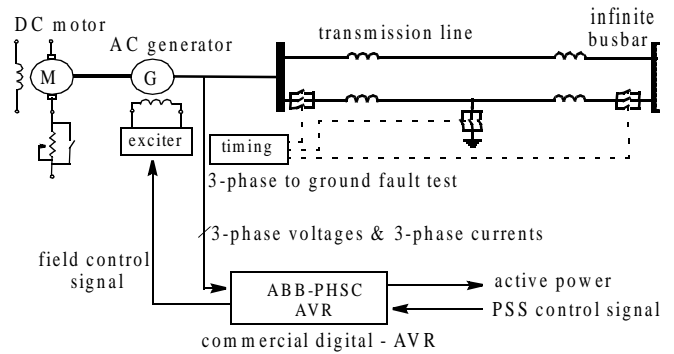


Fig. 2 Configuration of laboratory power system

converted by the digital to analogue converter (D/A). The calculation time of the OAPSS is less than 37ms.

The results are compared with those of the conventional IEEE type PSS1A power system stabilizer (CPSS) [14], implemented on the same DSP, with a 1 ms sampling period.

### 6.1 Voltage Reference Step Change

In this experiment the micro-synchronous generator was operated at 0.9 p.u. power, 0.85 power-factor lag. A 10% step increase in the reference voltage was applied at 2 s and removed at 8 s. The generator electrical power  $P_e$  with the proposed optimal adaptive power system stabilizer (OAPSS) and with the conventional power system stabilizer (CPSS) is shown in Fig. 3.

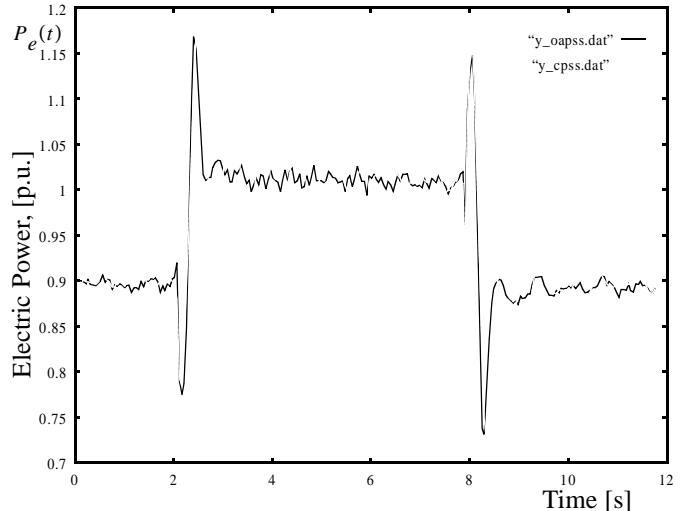


Fig. 3 Comparison of OAPSS and CPSS responses to a 10% step reference voltage disturbance at  $P_e = 0.9$  p.u.,  $\cos\phi = 0.85$  lag.

From the illustration in Fig. 3 it can be observed that the conventional CPSS controller damps very slowly as compared to the proposed OAPSS controller. The conventional controller exhibits an oscillatory damping which lasts for 3 s before dying out. This response can be seen at 2 s and at 8 s corresponding to the voltage input step. The proposed controller exhibits a faster and more controlled damping. The fluctuation in the generator electric power output is due to the random bit generator associated with the adaptive estimation algorithm

and it is not related to the control algorithm. Comparison of the control signals for OAPSS and CPSS is given in Fig. 4.

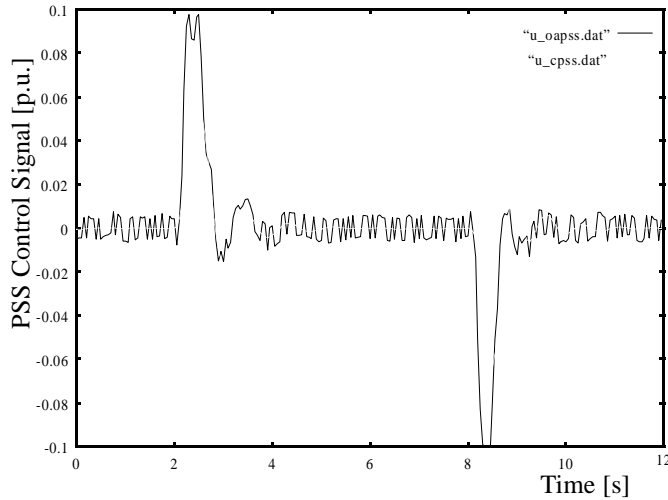


Fig. 4 Control signal of OAPSS and CPSS for a 10% step reference voltage disturbance at  $P_e = 0.9$  p.u.,  $\cos\phi=0.85$  lag

## 6.2 Input Torque Reference Step Change

In this experiment the generator was operated at  $0.9$  p.u. power,  $0.85$  power-factor lag. A  $0.4$  p.u. step decrease in the input torque was applied at  $2$  s and removed at  $8$  s. The generator electrical power  $P_e$  with the proposed optimal adaptive power system stabilizer (OAPSS) and with the conventional power system stabilizer (CPSS) is shown in Fig. 5.

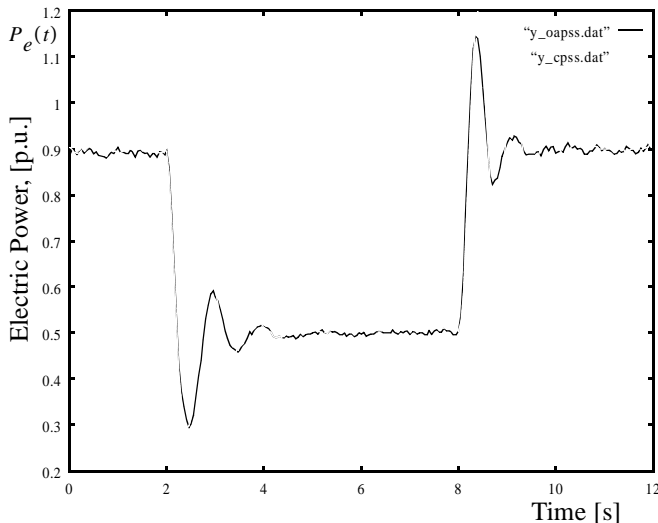


Fig. 5 Comparison of OAPSS and CPSS responses to a  $0.4$  p.u. step torque disturbance at  $P_e = 0.9$  p.u.,  $\cos\phi=0.85$  lag.

Differences in the performance of these two stabilizers can be seen in Fig. 5. When the generator condition changes to a lower operating point at  $2$  s both control algorithms provide good damping. However, when a  $0.4$  p.u. electric power step increase is applied to the system at  $8$  s, the system's stability margin is decreased due to the higher operating point and the performance with the OAPSS is considerably better.

## 6.3 Three-phase to Ground Fault Test

To investigate the performance of the OAPSS under transient conditions caused by transmission line fault, a three phase to ground fault test has been conducted. In this experiment the generator was operated at  $0.9$  p.u. power,  $0.85$  power-factor lag. At this operating condition, with both lines in operation, a three phase to ground fault was applied at  $2$  s in the middle of one transmission line. The faulty transmission line was opened by relay action at both ends  $100$  ms later. The first unsuccessful reclosure attempt was made after  $600$  ms, and the line was opened again  $100$  ms later due to a permanent fault. The second successful reclosure attempt was applied at  $8$  s and the system returned to the initial operating conditions.

System response with the OAPSS and the CPSS under the above transient conditions is shown in Fig. 6.

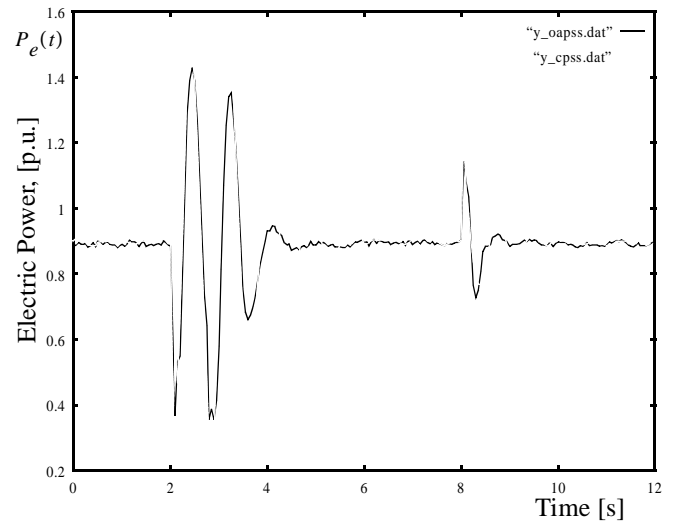


Fig. 6 Comparison of OAPSS and CPSS responses to a three phase to ground fault at  $P_e = 0.9$  p.u.,  $\cos\phi=0.85$  lag.

From Fig. 6 it can be observed that the OAPSS outperforms the CPSS with a smaller overshoot and faster settling time in both cases, at  $2$  s and  $8$  s. Fig. 7 presents the comparison of the performance in response to a three phase fault with the generator operating at  $0.5$  p.u. power and  $0.9$  power-factor lead.

One can observe that at  $2$  s the system response with the CPSS shows smaller oscillations, but these oscillations increase with time and push the system into an unstable oscillatory state up to  $8$  s, after which the second transmission line is restored. This test was repeated several times with the same outcome. This test has demonstrated that the system response with OAPSS is stable but with the CPSS is unstable. Comparison of the control signals for OAPSS and CPSS is given in Fig. 8.

## 7 Conclusions

The experimental identification studies, carried out on a micro-synchronous generator, show that an effective linear representation of the power-system dynamics around a particular operating point can be achieved with a reduced-order output-prediction structure. In particular the comparison between

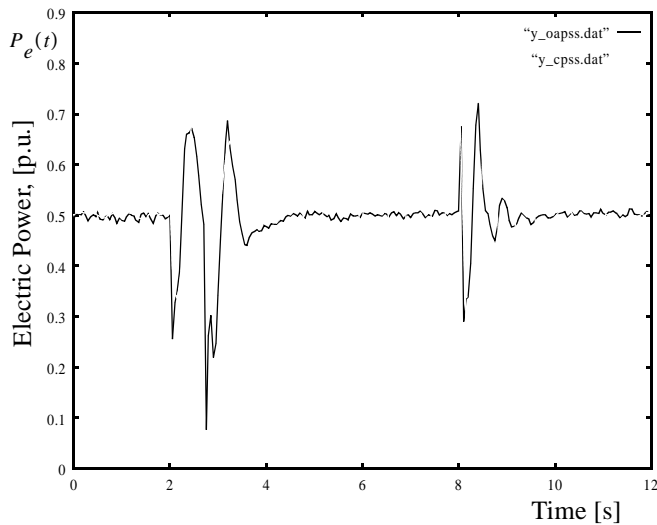


Fig. 7 Comparison of OAPSS and CPSS responses to a three phase to ground fault disturbance at  $P_e = 0.5$  p.u.,  $\cos\phi=0.9$  lead.

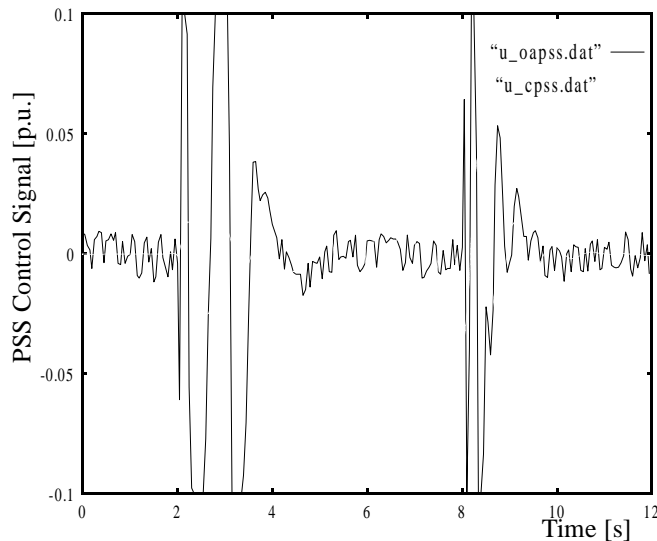


Fig. 8 Control signal of OAPSS and CPSS for a three phase to ground fault disturbance at  $P_e = 0.5$  p.u.,  $\cos\phi=0.9$  lead.

the actual system output changes and a one step ahead predicted output from the model is thought to be a particularly efficacious test of model validity. The test results provide considerable confidence in the effectiveness of the identified model.

Additional techniques were introduced to increase the algorithm's convergence and robustness. For fast system parameter tracking, an adaptive forgetting factor is used. The adaptive algorithm will recognize a sudden operating point change and will put more emphasis on recent measurement data by applying the adaptive forgetting factor. The required robustness is achieved by the implementation of a constraint function in the adaptive algorithm. This constraint is a function of the prediction error and prevents any sudden large changes in the identified system parameters, even if there is a large error in the prediction, caused by exogenous disturbances.

The states of the system are estimated by an adaptive Kalman filter, which is a novel approach, especially in the calculation of the state noise covariance. It is an alternative

approach to that of the conventional method of using a pre-defined measurement noise covariance and state noise covariance.

The idea described in this work effectively allows the implementation of optimal-control methods in a power system, while relaxing the requirement for access to the system states to merely those output variables which need to be directly controlled. The approach described is quite general and is therefore applicable to systems other than that studied here.

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